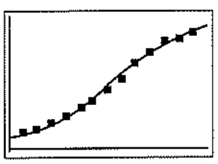
25. (a) $v(t) = 2t - 4$ (b) $a(t) = 2$
(c) It begins at position 3 moving in a negative direction. It moves to position -1 when $t = 2$, and then changes direction, moving in a positive direction thereafter.

27. (a) $v(t) = 3t^2 - 3$ (b) $a(t) = 6t$
(c) It begins at position 3 moving in a negative direction. It moves to position 1 when $t = 1$, and then changes direction, moving in a positive direction thereafter.

29. (a) $t = 2.2, 6, 9.8$ (b) $t = 4, 8, 11$

31. Some calculators use different logistic regression equations, so answers may vary.
(a) $y = \dfrac{12655.179}{1 + 12.871e^{-0.0326x}}$
(b)

[0, 140] by [−200, 12000]

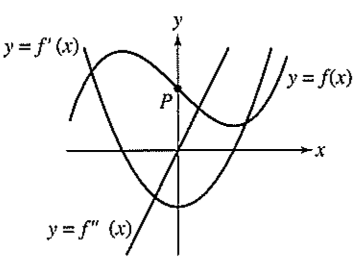
(c) The regression equation predicts a population of 12,209,870. (This is remarkably close to the 2000 census number of 12,281,054.)
(d) The second derivative has a zero at about 78, indicating that the population was growing the fastest in 1898. This corresponds to the inflection point on the regression curve.
(e) The regression equation predicts a population limit of about 12,655,179.

33. $y' = 3 - 3x^2$ and $y'' = -6x$.
$y' = 0$ at ± 1. $y''(-1) > 0$ and $y''(1) < 0$, so there is a local minimum at $(-1, 3)$ and a local maximum at $(1, 7)$.

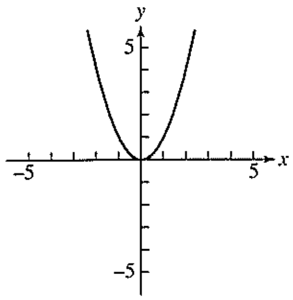
35. $y' = 3x^2 + 6x$ and $y'' = 6x + 6$.
$y' = 0$ at -2 and 0. $y''(-2) < 0$ and $y''(0) > 0$, so there is a local maximum at $(-2, 2)$ and a local minimum at $(0, -2)$.

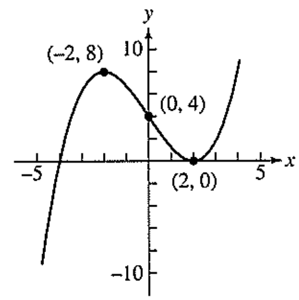
37. $y' = (x + 1)e^x$ and $y'' = (x + 2)e^x$.
$y' = 0$ at -1 and $y''(-1) > 0$, so there is a local minimum at $(-1, -1/e)$.

39. (a) None (b) At $x = 2$ (c) At $x = 1$ and $x = \dfrac{5}{3}$

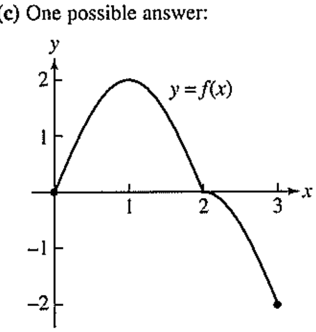
41.


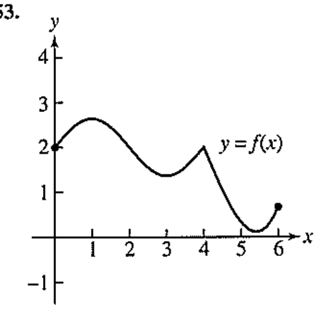
43. No. f must have a horizontal tangent line at that point, but it could be increasing (or decreasing) on both sides of the point, and there would be no local extremum.

45. One possible answer:


47. One possible answer:


49. (a) $[0, 1]$, $[3, 4]$, and $[5.5, 6]$
(b) $[1, 3]$ and $[4, 5.5]$
(c) Local maxima: $x = 1$, $x = 4$ (if f is continuous at $x = 4$), and $x = 6$; local minima: $x = 0$, $x = 3$, and $x = 5.5$

51. (a) Absolute maximum at $(1, 2)$; absolute minimum at $(3, -2)$
(b) None
(c) One possible answer:


53.


55. **False.** For example, consider $f(x) = x^4$ at $c = 0$.

57. A **59.** C

61. (a) In Exercise 7, $a = 4$ and $b = 21$, so $-\dfrac{b}{3a} = -\dfrac{7}{4}$, which is the x-value where the point of inflection occurs. The local extrema are at $x = -2$ and $x = -\dfrac{3}{2}$, which are symmetric about $x = -\dfrac{7}{4}$.
(b) In Exercise 2, $a = -2$ and $b = 6$, so $-\dfrac{b}{3a} = 1$, which is the x-value where the point of inflection occurs. The local extrema are at $x = 0$ and $x = 2$, which are symmetric about $x = 1$.
(c) $f'(x) = 3ax^2 + 2bx + c$ and $f''(x) = 6ax + 2b$. The point of inflection will occur where $f''(x) = 0$, which is at $x = -\dfrac{b}{3a}$.
If there are local extrema, they will occur at the zeros of $f'(x)$. Since $f'(x)$ is quadratic, its graph is a parabola and any zeros will be symmetric about the vertex, which will also be where $f''(x) = 0$.

63. (a) Since $f''(x)$ is quadratic it must have 0, 1, or 2 zeros. If $f''(x)$ has 0 or 1 zeros, it will not change sign and the concavity of $f(x)$ will not change, so there is no point of inflection. If $f''(x)$ has 2 zeros, it will change sign twice, and $f(x)$ will have 2 points of inflection.
(b) $f(x)$ has two points of inflection if and only if $3b^2 > 8ac$.

Quick Quiz (Sections 5.1–5.3)

1. C **3.** B

Section 5.4

Quick Review 5.4

1. None

3. $\frac{200\pi}{3}$ cm^3

5. $-\sin \alpha$ **7.** $\sin \alpha$

9. $x = 1$ and $y = \sqrt{3}$, or, $x = -1$ and $y = -\sqrt{3}$

Exercises 5.4

1. (a) As large as possible: 0 and 20; as small as possible: 10 and 10
(b) As large as possible: $\frac{79}{4}$ and $\frac{1}{4}$; as small as possible: 0 and 20

3. Smallest perimeter $= 16$ in., dimensions are 4 in. by 4 in.

5. (a) $y = 1 - x$
(b) $A(x) = 2x(1 - x)$
(c) Largest area $= \frac{1}{2}$, dimensions are 1 by $\frac{1}{2}$

7. Largest volume is $\frac{2450}{27} \approx 90.74$ in^3; dimensions: $\frac{5}{3}$ in. by $\frac{14}{3}$ in. by $\frac{35}{3}$ in.

9. Largest area $= 80{,}000$ m^2; dimensions: 200 m (perpendicular to river) by 400 m (parallel to river)

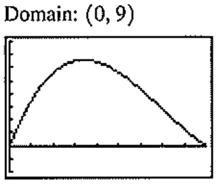
11. (a) 10 ft by 10 ft by 5 ft (b) Assume that the weight is minimized when the total area of the bottom and the 4 sides is minimized.

13. 18 in. high by 9 in. wide

15. $\theta = \frac{\pi}{2}$

17. $\frac{8}{\pi}$ to 1

19. (a) $V(x) = 2x(24 - 2x)(18 - 2x)$
(b) Domain: $(0, 9)$



[0, 9] by [–400, 1600]

(c) Maximum volume ≈ 1309.95 in^3 when $x \approx 3.39$ in.
(d) $V'(x) = 24x^2 - 336x + 864$, so the critical point is at $x = 7 - \sqrt{13}$, which confirms the result in part (c).
(e) $x = 2$ in. or $x = 5$ in.
(f) The dimensions of the resulting box are $2x$ in., $(24 - 2x)$ in., and $(18 - 2x)$ in. Each of these measurements must be positive, so that gives the domain of $(0, 9)$.

21. Dimensions: width ≈ 3.44, height ≈ 2.61; maximum area ≈ 8.98

23. Set $r'(x) = c'(x)$: $4x^{-1/2} = 4x$. The only positive critical value is $x = 1$, so profit is maximized at a production level of 1000 units. Note that $(r - c)''(x) = -2(x)^{-3/2} - 4 < 0$ for all positive x, so the Second Derivative Test confirms the maximum.

25. Set $c'(x) = c(x)/x$: $3x^2 - 20x + 30 = x^2 - 10x + 30$. The only positive solution is $x = 5$, so average cost is minimized at a production level of 5000 units. Note that $\frac{d^2}{dx^2}\left(\frac{c(x)}{x}\right) = 2 > 0$ for all positive x, so the Second Derivative Test confirms the minimum.

27. 67 people

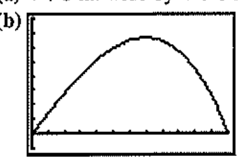
29. (a) $f'(x)$ is a quadratic polynomial, and as such it can have 0, 1, or 2 zeros. If it has 0 or 1 zeros, then its sign never changes, so $f(x)$ has no local extrema.
If $f'(x)$ has 2 zeros, then its sign changes twice, and $f(x)$ has 2 local extrema at those points.
(b) Possible answers:
No local extrema: $y = x^3$;
2 local extrema: $y = x^3 - 3x$

31. (a) $x = 12$ cm and $y = 6$ cm (b) $x = 12$ cm and $y = 6$ cm

33. (a) $a = 16$ (b) $a = -1$

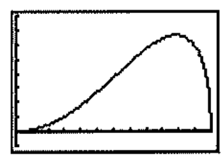
35. (a) $a = -3$ and $b = -9$ (b) $a = -3$ and $b = -24$

37. (a) $4\sqrt{3}$ in. wide by $4\sqrt{6}$ in. deep
(b)



[0, 12] by [−100, 800]

(c)



[0, 12] by [−100, 800]

Changing the value of k changes the maximum strength, but not the dimensions of the strongest beam. The graphs for different values of k look the same except that the vertical scale is different.

39. (a) Maximum speed $= 10\pi$ cm/sec;
maximum speed is at $t = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$ seconds;
position at those times is $s = 0$ cm (rest position); acceleration at those times is 0 cm/sec^2
(b) The magnitude of the acceleration is greatest when the cart is at positions $s = \pm 10$ cm; The speed of the cart is 0 cm/sec at those times.

41. The minimum distance is $\frac{\sqrt{5}}{2}$

43. No. It has an absolute minimum at the point $\left(\frac{1}{2}, \frac{3}{4}\right)$.

45. (a) Whenever t is an integer multiple of π sec
(b) The greatest distance is $3\sqrt{3}/2$ m when $t = 2\pi/3$ and $4\pi/3$ sec.

47. $\theta = \frac{\pi}{6}$

49. $M = \frac{C}{2}$

51. True. This is guaranteed by the Extreme Value Theorem (Section 5.1).

53. D **55.** B

57. Let P be the foot of the perpendicular from A to the mirror, and Q be the foot of the perpendicular from B to the mirror. Suppose the light strikes the mirror at point R on the way from A to B. Let:
a = distance from A to P
b = distance from B to Q
c = distance from P to Q
x = distance from P to R